CONSIDERATIONS REGARDING THE CORRELATION BETWEEN UWB ANTENNA TRANSMIT AND RECEIVE RESPONSES

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Abstract. Due to the reciprocity principle the (impulse) responses of antennas for transmission and reception are related by a temporal derivative. We provide some considerations and results regarding the impact of this relation on the shape of some elementary UWB signal waveforms that are to be transmitted or received.

INTRODUCTION

It is well known that the Lorentz reciprocity principle imposes a relation between the spatio-temporal transmit and receive characteristics of an antenna (cf. Kanda [1], Lo [2], Baum [3], Shlivinski et al. [4], Farr and Baum [5], Kunisch and Pamp [6]). In particular, for a given antenna the (temporal) impulse responses in transmission and reception are related by a time derivative. Thus, antennas generally exhibit differently shaped transmission and reception (impulse) responses to the same stimulus. The question arises as to how significant this difference is depending on the particular signal that is to be transmitted or received, and its relative bandwidth. The characterization of shape differences between the responses is useful for the design of UWB antennas that are used for both transmission and reception. For the modeling of UWB radio channels it is important to account for the dispersive properties of the involved antennas; a quantitative characterisation of the effect of the derivative relationship provides some guidance as to whether or not the antenna model (and hence the channel model) should account for this property e.g. for given accuracy requirements.

In the next sections, we will obtain a characterization of the effects of the derivative relationship by applying a correlation measure for some example stimuli.

ANTENNA RESPONSES

Following the notation given in Kunisch and Pamp [6], a transmitting antenna may be described by its spatio-temporal transmit characteristic $\mathbf{A}\left(\hat{\mathbf{r}}_{12},\omega\right)$ defined by the electric far field $\mathbf{E}_1\left(\mathbf{r}_2,\omega\right)=a\left(\omega\right)\mathbf{A}\left(\hat{\mathbf{r}}_{12},\omega\right)\sqrt{Z_0}\,\mathrm{e}^{-jk_0r_{12}}\big/\sqrt{4\pi}r_{12}$ at $\mathbf{r}_2$ produced by the antenna at $\mathbf{r}_1$ at angular frequency ω in direction $\hat{\mathbf{r}}_{12}\triangleq\left(\mathbf{r}_2-\mathbf{r}_1\right)\big/\left\|\mathbf{r}_2-\mathbf{r}_1\right\|$; $a\left(\omega\right)$ is the wave incident via the feed line, and Z_0 is the free-space impedance. Similarly, the spatio-temporal receive characteristic $\mathbf{h}\left(\hat{\mathbf{k}},\omega\right)$ of the antenna is defined by $b^i\left(\omega\right)=\sqrt{4\pi}\mathbf{h}\left(\hat{\mathbf{k}},\omega\right)\mathbf{E}^i\left(\mathbf{r}_1,\hat{\mathbf{k}},\omega\right)\big/\sqrt{Z_0}$, where the receiving antenna acts upon its feed line as a wave source of strength b^i for an incident plane wave field $\mathbf{E}^i\left(\mathbf{r}_1,\hat{\mathbf{k}},\omega\right)$ propagating in direction $\hat{\mathbf{k}}$. For a given $\hat{\mathbf{k}}$, the antenna acts as a temporal filter that responds to a stimulus $s\left(\omega\right)$ with $\tilde{r}^T\left(t\right)\triangleq\int s\left(\omega\right)\mathbf{A}\left(\omega\right)\exp(j\omega t)\,\mathrm{d}\omega$ in transmission and $\tilde{r}^R\left(t\right)\triangleq\int s\left(\omega\right)\mathbf{h}\left(\omega\right)\exp(j\omega t)\,\mathrm{d}\omega$ in reception. [1] Furthermore, using Lorentz reciprocity, it can be shown that under quite general conditions the transmit and receive characteristics are related by $2j\omega\mathbf{h}\left(-\hat{\mathbf{k}},\omega\right)=c_0\mathbf{A}\left(\hat{\mathbf{k}},\omega\right)$. Therefore, for transmission we also have[2]

$$\tilde{r}^T\left(t\right)\propto\int s\left(\omega\right)j\omega\mathbf{h}\left(\omega\right)\exp(j\omega t)\,\mathrm{d}\omega=\frac{\partial}{\partial t}\tilde{r}^R\left(t\right), \tag{1}$$

i.e., the transmission response is proportional to the temporal derivative of the receive response. Consequently, the transmit and receive responses to the same stimulus $s\left(\omega\right)$ will exhibit different shapes. This means that, for example, an "impulse radiating" antenna is not necessarily an "impulse receiving" antenna.

If the considered stimuli $s\left(\omega\right)$ are restricted to be band-limited, i.e., non-zero only for

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[1] The tilde ~ is used here to denote time dependent quantities.

[2] To simplify the notation, the dependency on $\hat{\mathbf{k}}$ will not be given explicitly in the remainder.

$\omega \in B(\Omega,\omega_c) \triangleq \{\omega : \left||\omega| - \omega_c\right| \le \Omega/2\}$, $\omega_c, \Omega > 0$, then Eq. (1) may be rewritten as

$$\tilde{r}^T(t) \propto (\partial/\partial t)\tilde{r}^R(t) = \int_{\omega\in B} j\omega s(\omega)\mathbf{h}(\omega)\exp(j\omega t)\mathrm{d}\omega = \int_{\omega\in B} |\omega| s(\omega)\mathbf{h}(\omega)\exp\left(j\omega\left(t+\tau(\omega)\right)\right)\mathrm{d}\omega \qquad (2)$$

where $\tau(\omega) \triangleq \pi/(2|\omega|)$. Furthermore, in the narrowband case we have $\Omega \ll \omega_c$; hence approximations $|\omega| \approx \omega_c$, $\tau(\omega) \approx \pi/(2\omega_c)$ may be used yielding

$$\tilde{r}^T(t) \propto (\partial/\partial t)\tilde{r}^R(t) \approx \omega_c \int_{\omega\in B} s(\omega)\mathbf{h}(\omega)\exp\left(j\omega\left(t+\tau(\omega_c)\right)\right)\mathrm{d}\omega \propto \tilde{r}^R\left(t+\frac{\pi}{2\omega_c}\right). \qquad (3)$$

Therefore, in the low relative bandwidth limit, the antenna responses in transmission and reception are approximately equal up to scaling and shifting; in other words, the derivative operator applied to "sufficiently" narrowband signals approximately just scales and shifts its operand. In the extreme case of a CW signal $\tilde{r}^R(t) = \cos(\omega_c t + \phi)$, in fact $(\partial/\partial t)\tilde{r}^R(t) \propto r^R\left(t + (\pi/2\omega_c)\right)$ is not an approximation, but exact. In general, however, the accuracy of approximation (3) will depend on the actual relative bandwidth Ω/ω_c and the particular stimulus $s(\omega)$ under consideration.

SHAPE DIFFERENCE MEASURE

To assess the shape differences quantitatively, we consider the maximum absolute value of the correlation coefficient of the receive response and its derivative,[3]

$$\rho\left(\tilde{r}^R\right) \triangleq \max_\tau \frac{\left|\int\left(\partial\tilde{r}^R/\partial t\right)(t)\,\tilde{r}^R(t+\tau)\mathrm{d}t\right|}{\sqrt{\int\left[\left(\partial\tilde{r}^R/\partial t\right)(t)\right]^2\mathrm{d}t\int\left[\tilde{r}^R(t)\right]^2\mathrm{d}t}}. \qquad (4)$$

Note that $\rho = 1$ for CW signals. In the next section, we will consider some example stimuli for two ideal cases: constant gain and phase ($\mathbf{A}(\omega) = \mathrm{const}$), and constant effective length ($\mathbf{h}(\omega) = \mathrm{const}$).

RESULTS

As first example for a stimulus we consider the *n*-th derivative of the Gaussian impulse, $s_G(\omega,n) \triangleq (j\omega)^n \exp(-a\omega^2)$. Note that ρ is independent of the factor *a*. Figure 1 (top row) shows ρ as function of *n*. The decorrelation is generally larger in the $\mathbf{A}(\omega) = \mathrm{const}$ case because for $\mathbf{A}(\omega) = \mathrm{const}$ the mapping from stimulus to response involves one derivative less for both transmission and reception compared to the $\mathbf{h}(\omega) = \mathrm{const}$ case, thus resulting in smoother responses. Consequently, both curves are equal up to a shift of the argument *n* by 1. For the Gaussian impulse, the decorrelation for *n* = 1 caused by changing from $\mathbf{h}(\omega) = \mathrm{const}$ to $\mathbf{A}(\omega) = \mathrm{const}$ amounts to almost 0.2. For both ideal antennas, $\rho > 0.8$ for n ≥ 2. The curve can be approximated reasonably by a fourth-order polynomial as shown for $\mathbf{h}(\omega) = \mathrm{const}$. For n < 1 and $\mathbf{A}(\omega) = \mathrm{const}$, $\mathbf{h}(\omega)$ exhibits a singularity at $\omega = 0$; therefore this range is excluded here.

The bottom row in Figure 1 shows ρ as function of the relative bandwidth Ω/ω_c for three types of signals: the curve labelled GN gives the expected value of ρ for band-limited white Gaussian noise (with power spectral density constant in $B(\Omega,\omega_c)$ and zero elsewhere); curve BP corresponds to an ideal bandpass signal ($s_{BP}(\omega) = 1$ for $\omega \in B(\Omega,\omega_c)$, zero elsewhere); and curve GS pertains to a sinusoid with Gaussian envelope $\tilde{s}_{GS}(t) \triangleq \exp\left(-(\Omega t/2\pi)^2\right)\sin(\omega_c t)$, where Ω corresponds roughly to the – 20 dB bandwidth.

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[3] The integrals extend over the respective integrand supports. For non-finite energy integrands, read $\int$ as $\lim_{T\to\infty}\frac{1}{2T}\int_{[-T,T]}$.

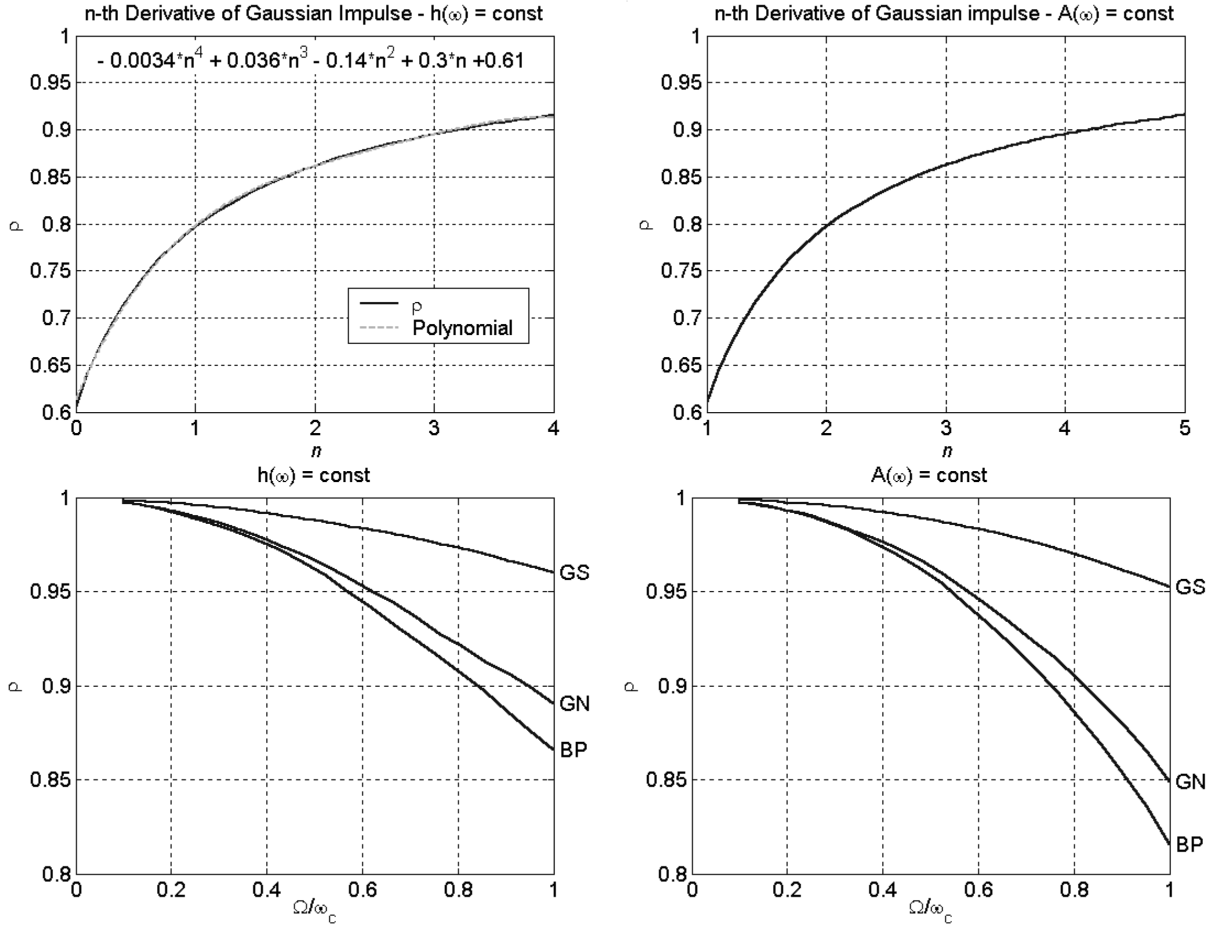

Figure 1. *Variation of* ρ.

The GS curve is also valid for the incoherent case where $\tilde{s}_{GS}(t) \triangleq \exp\left(-(\Omega t/2\pi)^2\right)\sin(\omega_c t + \varphi)$ with a uniformly distributed random phase φ; the curve then gives the expected value of ρ. In all these cases we find that $\rho > 0.8$, with larger decorrelation for $\mathbf{A}(\omega) = \text{const}$.

CONCLUSIONS

For some elementary waveforms used as stimulus we have considered the maximum correlation between the corresponding antenna responses in transmission and reception for the two ideal cases of antennas with constant gain and phase, and constant effective length. Although there is an additional derivative in the transmit response, the shape differences between the two responses can be low enough to allow for maximum correlation values larger than 0.8 for UWB relative bandwidths up to 1, and larger than 0.95 at the lower UWB range with relative bandwidths around 0.2 or 0.25.

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