Transmit Reference Impulse Radio Systems using Weighted Correlation

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***Abstract*— We propose a generalization of the transmit-reference (TR) communication scheme by employing a weighted correlation stage at the demodulator. A statistical characterization and a method to evaluate the bit error probability (BEP) of the generalized TR systems is presented, in the presence of multipath, inter/intra symbol interference and multi-user interference, for any type of front-end filter and weighting function. Two criteria are considered to shape the weighting function: the linear minimum mean square error (LMMSE) criterion and the maximum ratio combining (MRC) criterion. For both criteria, the BEP is presented for a single user, AWGN channel.**

***Index Terms*— UWB, Indoor Communication, delay hopped transmit reference, weighted correlation**

I. Introduction

Since FCC's R&O [1], the interest in ultra wideband (UWB) technology has grown continuously. One of the technologies to obtain UWB signals is impulse radio (IR) [2], where ultra-short pulses, called monocycles, are transmitted. The radiation of IR signals is relatively simple, enabling the transmitter to be small and low cost, see [3]. However, the typical impulse response of an indoor radio channel contains significant amount of multipath components [4]. Consequently, a coherent receiver has to sample the received signal at a GHz rate or requires many rake-fingers. Either way, the receiver will be complex and costly.

A practical, sub-optimal communication system, based on the transmit reference (TR) principle, is proposed in [5]. In the basic scheme, pulses are transmitted in pairs, where the first pulse is unmodulated and the second is modulated by data. If the delay between both pulses is small compared to the coherence time of the channel, both pulses are distorted in the same way by the channel, such that the first pulse can be used as reference for the demodulation of the second pulse. The advantage is that a TR-receiver needs no channel estimation and allows for a simplified synchronization. The drawback is a 6 dB loss compared to a perfect matched filter receiver (3 dB due to corruption of the reference and 3 dB due to usage of two pulses per bit). The loss significantly diminish when the TR-receiver is compared in terms of performance with sub-optimal Rake receivers, equipped with a limited number of fingers and with an imperfect channel state information at the receiver, see [9]. A further loss reduction can be obtained by sending additional modulated pulses per reference pulse. Additionally, the receiver delay must be well-matched to the transmitter delay, which from an implementation point of view is challenging. Based on the pro's and con's, the TR principle is most likely not suitable for high-data rate applications, but it is a candidate for low-cost, low-data-rate applications.

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In the system proposed by [5], the receiver antenna output signal is multiplied with a delayed version of itself, followed by an integration of the multiplier output. Consequently, no weighting is applied to multiplier output, although the signal to interference and noise ratio (SINR) of the multiplier output may vary in time. Therefore, we propose the usage of a weighted correlation stage at the demodulator, to improve the performance of the TR principle.

The goal of this paper is to present a mathematical frame work that allows for a closed-form computation of the bit error probability, taking into account the radio channel, the transmitter and receiver front-end filters, inter symbol interference and multi-user interference. The weighted correlator is a generalization of the normal correlator, such that the framework can also be used for the TR system proposed in [5]. The paper organization is as follows. In section II, the system model will be introduced. In the following section, a discrete time equivalent model of the system will be developed, enabling the statistical characterization of the system (sec. IV). The characterization allows for a bit error probability (BEP) computation (sec. V) and determination of the weighting function (sec. VI). Preliminary results will be presented, followed by the conclusions.

II. System model

In this section, the principal characteristics of the system will be described. The time axis is divided into frames of T_s seconds, which are further subdivided into N_h intervals called chips of duration T_c seconds. For simplicity, we will assume that $N_h T_c = T_s$. Within each frame, two identically shaped pulses are transmitted, a reference pulse $x(t)$ followed by a modulated one. Let's focus on the waveform transmitted within the j-th frame by the user k. The reference pulse is positioned in the chip $c_j^{(k)}$ and the modulated pulse is transmitted $d_j^{(k)}$ chips late. Notice that both $c_j^{(k)}$ and $d_j^{(k)}$ may vary on a frame by frame base and hence a merger of time-hopping (TH) and delay-hopping (DH). The TH code associated to the j-th frame of user k written as a column vector is $\mathbf{s}_j^{(k)} = [s_{j,i}^{(k)}]$ where

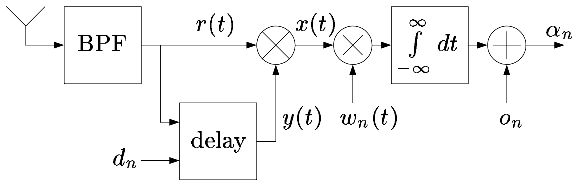

Fig. 1. Block diagram of the general weighted correlator

$s_{j,i}^{(k)} = \delta[i - c_j^{(k)}]$ for $i = 1, 2, ...N_h$ and $\delta[i]$ denotes the Kronecker delta. Similarly, the DH and TH will be joint in a column vector denoted by $\tilde{\mathbf{s}}_j^{(k)} = [\tilde{s}_{j,i}^{(k)}]$ where $\tilde{s}_{j,i}^{(k)} = \delta[i - c_j^{(k)} - d_j^{(k)}]$ for $i = 1, 2, ...N_h$. We limit our analysis to BPSK modulation, such that $b_j^{(k)} \in \mathbb{B}$, where $\mathbb{B} = \{-1, 1\}$, both events having the same probability. The waveform transmitted in the j-th frame becomes

$$p_j^{(k)}(t) = \left(\mathbf{s}_j^{(k)} + b_j^{(k)} \tilde{\mathbf{s}}_j^{(k)}\right)^T \mathbf{v}(t), \quad (1)$$

where

$$\mathbf{v}(t) = [v(t), v(t - T_c), \ldots, v(t - (N_h - 1)T_c)]^T. \quad (2)$$

The signal transmitted by user k becomes

$$s^{(k)}(t) = \sum_j p_j^{(k)}(t - jT_s). \quad (3)$$

At the receiver, the incoming signal passes through a front-end filter to suppress out-of-band noise and interference. Our system deploys a band-pass filter (BPF); however, the results we present are valid for all filter types. Assuming that N_u users are active, the BPF output signal can be written as

$$r(t) = \sum_{k=1}^{N_u} \sqrt{A_k} s^{(k)}(t) * h^{(k)}(t) * p(t) + n(t), \quad (4)$$

where $p(t)$ is the BPF impulse response, the set $\{A_k\}$ contains the attenuations due to path loss and $n(t)$ is a Gaussian noise process, obtained by band-pass filtering a white Gaussian noise process with two-sided power spectral density $N_0/2$. In the rest of the paper, we will use $q^{(k)}(t)$ to denote the convolution between the transmitted pulse $v(t)$, the channel impulse response $h^{(k)}(t)$ and the BPF impulse response $p(t)$.

Similar to [5], the BPF output is correlated with a delayed version of itself; however, in our analysis a weighted correlation is applied. The delay is chosen such that the reference pulse is correlated with the modulated pulse. The decision statistic corresponding to the n-th bit of the user 1, α_n can be written as

$$\alpha_n = \int_{-\infty}^{\infty} r(t) r(t - d_n^{(1)} T_c) w_n(t) dt + o_n. \quad (5)$$

The window function $w_n(t)$ can be used to mimic any linear weighting applied on the multiplier output and o_n is a scalar to compensate for any offset in the demodulator output, before a decision on the bit value is made. A graphical representation of the demodulator structure can be found in Fig.(1).

III. Discrete time equivalent model

In order to analytically evaluate the performance in terms of the BEP, a discrete time equivalent model is constructed, obtained by sampling $r(t)$ every T_r seconds, where T_r is chosen to be at least the Nyquist rate. Without loss of generality, the bit under demodulation is consideration to be transmitted within the n-th frame by user 1. The corresponding weighting function $w_n(t)$ is assumed to be zero outside a specific observation interval $[0, T_{ob}]$, where T_{ob} denotes the duration of the observation interval. The vector containing the samples of $r(t)$ relevant to the demodulation is

$$\mathbf{r}_n = [r(nT_s), r(T_r + nT_s), \ldots, r((N_{ob} - 1)T_r + nT_s)]^T \quad (6)$$

To construct the discrete time equivalent model, we define several parameters. For simplicity, the user specific parameters are assume to be the same for all users. The enumeration of the parameters is

- N_l; number of bits of an individual user with a frame-index smaller than n that influence the received signal within the observation interval,
- N_r; number of bits of an individual user with a frame-index larger than n that influence the received signal within the observation interval,
- $N_t = N_r + N_l + 1$; total number of bits of an individual user that influence the received signal within the observation interval,
- $N_{ob} = T_{ob}/T_r$; the observation interval duration expressed in samples,
- $N_d = d_n^{(1)} T_c / T_r$; the delay of the modulated pulse under consideration, expressed in samples,
- $N_b = N_u N_t$; the total number of bits that influence the received signal within the observation interval,
- $L_b = N_l + 1$; the position of the bit under consideration within the N_t bits of user 1.

Furthermore, we define a noise vector denoted by $\mathbf{n}_n = [n_{nj}]$, where $n_{nj} = n(jT_r + nT_s)$ for $j = -N_d, 1 - N_d, \ldots, N_{ob} - 1$. Compared to $\mathbf{r}_n$, $\mathbf{n}_n$ contains additional N_d elements, to simplify the derivation. The elements of the noise vector are stationary, Gaussian distributed random variables (RVs), characterized by the discrete autocorrelation function

$$r_{nn}[k] = \frac{1}{2} N_0 p[k] * p[-k], \quad (7)$$

where $p[k]$ denotes the time discrete impulse response of BPF.

A user-specific TH-DH block diagonal matrix $\tilde{\mathbf{S}}_n^{(k)}$ is defined, where k is the user identifier. The TH-DH matrix is constructed as

$$\tilde{\mathbf{S}}_n^{(k)} = \mathbf{I}_{N_t, N_t} \odot \begin{bmatrix} \tilde{\mathbf{s}}_{-N_l+n}^{(k)} & \tilde{\mathbf{s}}_{-N_l+n+1}^{(k)} & \ldots & \tilde{\mathbf{s}}_{N_r+n}^{(k)} \end{bmatrix} \quad (8)$$

where $\odot$ denotes the Khatri-Rao product (column-wise Kronecker product), such that $\mathbf{S}_n^{(k)} \in \mathbb{R}^{N_h N_t, N_t}$. Similarly, a user-specific TH block diagonal matrix is introduced as follows

$$\mathbf{S}_n^{(k)} = \mathbf{I}_{N_t, N_t} \odot \begin{bmatrix} \mathbf{s}_{-N_l+n}^{(k)} & \mathbf{s}_{-N_l+n+1}^{(k)} & \ldots & \mathbf{s}_{N_r+n}^{(k)} \end{bmatrix} \quad (9)$$

Furthermore, an elementary chip delay matrix $\mathbf{D}$ is defined, where $\mathbf{D} = [d_{ij}]$ with $d_{ij} = \delta[i - j - 1]$ for $i, j = 0, 1, \ldots, N_t N_h - 1$. Finally, we define a user-specific channel matrix $\mathbf{Q}^{(k)} = [q_{ij}]$ with $q_{ij} = q^{(k)}(N_l T_s - iT_c + jT_r)$, for $i = 0, 1, \ldots, N_{ob} - 1$ samples and $j = 0, 1, \ldots, N_t N_h - 1$ chips, such that the channel matrix $\mathbf{Q}^{(k)} \in \mathbb{R}^{N_{ob}, N_t N_h}$. Based on these definitions, the received signal can be written as

$$\mathbf{r}_n = \sum_{k=1}^{N_u} \mathbf{Q}^{(k)} \left(\mathbf{S}_n^{(k)} \mathbf{1} + \tilde{\mathbf{S}}_n^{(k)} \mathbf{b}_n^{(k)} \right) + \mathbf{W}_1 \mathbf{n}_n, \tag{10}$$

where N_u denotes the number of interfering users, $\mathbf{1}$ is an all 1's column vector, $\mathbf{b}_n^{(k)}$ denotes a bit vector, where $\mathbf{b}_n^{(k)} = [b_{n+j}^{(k)}]$ for $j = -N_l, 1-N_l, \ldots, N_r$. Finally, the matrix $\mathbf{W}_1 = [w_{ij}]$ with $w_{ij} = \delta[i - j - N_d]$ for $i = 1, 2, \ldots, N_{ob}$ and $j = 1, 2, \ldots, N_{ob} + N_d$. The notation for the received signal can be further reduced to

$$\mathbf{r}_n = \mathbf{R}_n^{(0)} \mathbf{1} + \tilde{\mathbf{R}}_n^{(0)} \mathbf{b}_n + \mathbf{W}_1 \mathbf{n}_n, \tag{11}$$

where

$$\mathbf{R}_n^{(d)} = \left[\mathbf{Q}^{(1)} \mathbf{D}^d \mathbf{S}_n^{(1)}, \mathbf{Q}^{(2)} \mathbf{D}^d \mathbf{S}_n^{(2)}, \cdots \mathbf{Q}^{(N_u)} \mathbf{D}^d \mathbf{S}_n^{(N_u)} \right] \tag{12}$$

$$\tilde{\mathbf{R}}_n^{(d)} = \left[\mathbf{Q}^{(1)} \mathbf{D}^d \tilde{\mathbf{S}}_n^{(1)}, \mathbf{Q}^{(2)} \mathbf{D}^d \tilde{\mathbf{S}}_n^{(2)}, \cdots \mathbf{Q}^{(N_u)} \mathbf{D}^d \tilde{\mathbf{S}}_n^{(N_u)} \right] \tag{13}$$

$$\mathbf{b}_n = \left[\left(\mathbf{b}_n^{(1)} \right)^T, \left(\mathbf{b}_n^{(2)} \right)^T \cdots \left(\mathbf{b}_n^{(N_u)} \right)^T \right]^T. \tag{14}$$

The delayed version of the received signal can be written as

$$\mathbf{y}_n = \mathbf{R}_n^{(d_n^{(1)})} \mathbf{1} + \tilde{\mathbf{R}}_n^{(d_n^{(1)})} \mathbf{b}_n + \mathbf{W}_2 \mathbf{n}_n, \tag{15}$$

where $\mathbf{W}_2 = [w_{ij}]$ with $w_{ij} = \delta_{i-j}$ for $i = 1, 2, \ldots, N_{ob}$ and $j = 1, 2, \ldots, N_{ob} + N_d$. The multiplier output in a vectorial notation becomes

$$\mathbf{x}_n = \mathbf{\Lambda}(\mathbf{r}_n)\, \mathbf{y}_n, \tag{16}$$

where $\mathbf{\Lambda}(.)$ denotes the diagonal operator. Substitution of Eq.(11) and Eq.(15) into Eq.(16) leads to the following expression for

$$\begin{aligned}\mathbf{x}_n = &\mathbf{\Lambda}\left(\mathbf{R}_n^{(0)} \mathbf{1} \right) \mathbf{R}_n^{(d_n^{(1)})} \mathbf{1} + \mathbf{C}_n \mathbf{b}_n + \sum_{k=1}^{N_b} \mathbf{K}_{nk} \mathbf{b}_n \left[\mathbf{b}_n \right]_k \\ &+ \mathbf{T}_n \mathbf{n}_n + \mathbf{z}_n,\end{aligned} \tag{17}$$

where $[\mathbf{a}]_k$ denotes the k-th element of the vector $\mathbf{a}$. Furthermore

$$\mathbf{C}_n = \mathbf{\Lambda}\left(\mathbf{R}_n^{(0)} \mathbf{1} \right) \tilde{\mathbf{R}}_n^{(d_n^{(1)})} + \mathbf{\Lambda}\left(\mathbf{R}_n^{(d_n^{(1)})} \mathbf{1} \right) \tilde{\mathbf{R}}_n^{(0)}, \tag{18}$$

$$\mathbf{T}_n = \mathbf{c}_n + \sum\nolimits_{k=1}^{N_b} \mathbf{L}_{nk} \left[\mathbf{b}_n \right]_k, \tag{19}$$

$$\mathbf{c}_n = \mathbf{\Lambda}\left(\mathbf{R}_n^{(0)} \mathbf{1} \right) \mathbf{W}_2 + \mathbf{\Lambda}\left(\mathbf{R}_n^{(d_n^{(1)})} \mathbf{1} \right) \mathbf{W}_1, \tag{20}$$

$$\mathbf{L}_{nk} = \mathbf{\Lambda}\left(\tilde{\mathbf{R}}_n^{(0)} \mathbf{e}_k \right) \mathbf{W}_2 + \mathbf{\Lambda}\left(\tilde{\mathbf{R}}_n^{(d_n^{(1)})} \mathbf{e}_k \right) \mathbf{W}_1, \tag{21}$$

$$\mathbf{K}_{nk} = \mathbf{\Lambda}\left(\tilde{\mathbf{R}}_n^{(0)} \mathbf{e}_k \right) \tilde{\mathbf{R}}_n^{(d_n^{(1)})}, \tag{22}$$

$$\mathbf{z}_n = \mathbf{\Lambda}(\mathbf{W}_1 \mathbf{n}_n)\, \mathbf{W}_2 \mathbf{n}_n, \tag{23}$$

where $\mathbf{e}_k$ is a column vector with $[\mathbf{e}_k]_l = \delta_{k-l}$ for $l = 1, 2, \ldots, N_b$.

The discrete time equivalent for the decision statistics variable α_n is given by

$$\alpha_n = \mathbf{w}_n^T \mathbf{x}_n + o_n, \tag{24}$$

where $\mathbf{w}_n$ is a vector representing the discrete time equivalent of the weighting function.

IV. Statistical characterization

A. Multiplier output

In this section we present a statistical characterization of the multiplier output, relevant for the derivation of the weighting function and for the computation of BEP. The derivations will be kept brief, due to space restrictions. The notation will be as follows: a mathematical expectation is denoted by $\mathcal{E}[.]$ and the covariance by $\mathcal{C}[.,.]$, where $\mathcal{C}\left[\mathbf{a}, \mathbf{b}^T\right] \triangleq \mathcal{E}\left[\mathbf{a}\mathbf{b}^T\right] - \mathcal{E}[\mathbf{a}_n]\, \mathcal{E}\left[\mathbf{b}_n^T\right]$. To shorten the notation further, define the autocovariance as $\mathcal{A}[\mathbf{a}] \triangleq \mathcal{C}\left[\mathbf{a}, \mathbf{a}^T\right]$.

The covariance vector containing the statistical correlation between the bit under demodulation $b_n^{(1)}$ and the multiplier output $\mathbf{x}_n$ is found to be

$$\mathcal{C}\Big[[\mathbf{b}_n]_{L_b}, \mathbf{x}_n \Big] = \mathbf{C}_n \mathcal{E}\Big[\mathbf{b}_n [\mathbf{b}_n]_{L_b} \Big] = \mathbf{C}_n \mathbf{e}_{L_b}, \tag{25}$$

since $\mathcal{E}\left[[\mathbf{b}_n]_k [\mathbf{b}_n]_l [\mathbf{b}_n]_m \right] = 0$ for all k, l, m.

The expectation of $\mathbf{x}_n$ is given by

$$\begin{aligned}\mathcal{E}[\mathbf{x}_n] = &\mathbf{\Lambda}\left(\mathbf{R}_n^{(0)} \mathbf{1} \right) \mathbf{R}_n^{(d_n^{(1)})} \mathbf{1} + \mathcal{E}[\mathbf{T}_n]\, \mathcal{E}[\mathbf{n}_n] + \mathcal{E}[\mathbf{z}_n] \\ &+ \mathbf{C}_n \mathcal{E}[\mathbf{b}_n] + \sum_{k=1}^{N_b} \mathbf{K}_{nk} \mathcal{E}\left[\mathbf{b}_n [\mathbf{b}_n]_k \right].\end{aligned} \tag{26}$$

Taking an element-wise view on the last expectation makes its solution obvious, since

$$\left[\mathcal{E}[\mathbf{z}_n] \right]_i = \mathcal{E}[n[i] n[i - N_d]] = r_{nn}[N_d], \tag{27}$$

which combined with $\mathcal{E}[\mathbf{b}_n] = 0$, $\mathcal{E}\left[\mathbf{b}_n [\mathbf{b}_n]_k \right] = \mathbf{e}_k$ and $\mathcal{E}[\mathbf{n}_n] = 0$ leads to

$$\mathcal{E}[\mathbf{x}_n] = \mathbf{\Lambda}\left(\mathbf{R}_n^{(0)} \mathbf{1} \right) \mathbf{R}_n^{(d_n^{(1)})} \mathbf{1} + \sum_{k=1}^{N_b} \mathbf{K}_{nk} \mathbf{e}_k + r_{nn}[N_d] \mathbf{1} \tag{28}$$

The signal $\mathbf{x}_n$ consists of 4 random terms meaning that its autocovariance matrix, before simplification, will contain 16 terms. Most of these terms are zero due to one of the following two reasons; a) statistical independent components or b) since all the odd moment of evenly distributed RV are zero. A straight-forward derivation leads to

$$\begin{aligned}\mathcal{A}[\mathbf{x}_n] = &\mathcal{A}[\mathbf{C}_n \mathbf{b}_n] + \mathcal{A}\left[\left(\sum_{k=1}^{N_b} \mathbf{K}_{nk} \mathbf{b}_n [\mathbf{b}_n]_k \right) \right] \\ &+ \mathcal{A}[\mathbf{T}_n \mathbf{n}_n] + \mathcal{A}[\mathbf{z}_n].\end{aligned} \tag{29}$$

The first term on the right-hand side of Eq.(29) is

$$\mathcal{A}[\mathbf{C}_n \mathbf{b}_n] = \mathbf{C}_n \mathcal{E}\left[\mathbf{b}_n \mathbf{b}_n^T \right] \mathbf{C}_n^T = \mathbf{C}_n \mathbf{I} \mathbf{C}_n^T = \mathbf{C}_n \mathbf{C}_n^T. \tag{30}$$

The second autocovariance matrix can be found to be

$$\begin{aligned}\mathcal{A}\left[\left(\sum_{k=1}^{N_b} \mathbf{K}_{nk} \mathbf{b}_n [\mathbf{b}_n]_k \right) \right] = &\sum_{k=1}^{N_b} \mathbf{K}_{nk} \left(\mathbf{I} - 2\mathbf{e}_k \mathbf{e}_k^T \right) \mathbf{K}_{nk}^T \\ &- \sum_{k,l=1}^{N_b} \mathbf{K}_{nk} \left(\mathbf{e}_k \mathbf{e}_l^T \right) \mathbf{K}_{nl}^T.\end{aligned} \tag{31}$$

The third covariance term is given by $\mathcal{A}[\mathbf{T}_n\mathbf{n}_n] =$

$$\mathcal{E}\left[\left(\mathbf{c}_n + \sum_{k=1}^{N_b} [\mathbf{b}_n]_k \mathbf{L}_{nk}\right) \mathbf{n}_n\mathbf{n}_n^T \left(\mathbf{c}_n^T + \sum_{l=1}^{N_b} [\mathbf{b}_n]_k \mathbf{L}_{nl}^T\right)\right]$$

$$= \mathbf{c}_n\mathcal{E}\left[\mathbf{n}_n\mathbf{n}_n^T\right]\mathbf{c}_n^T + \sum_{k,l=1}^{N_b}\mathcal{E}\left[[\mathbf{b}_n]_k[\mathbf{b}_n]_l\right] \mathbf{L}_{nk}\mathcal{E}\left[\mathbf{n}_n\mathbf{n}_n^T\right] \mathbf{L}_{nl}^T \quad (32)$$

Since $\mathcal{E}\left[[\mathbf{b}_n]_k [\mathbf{b}_n]_l\right] = \delta_{k-l}$, we obtain

$$\mathcal{A}[\mathbf{T}_n\mathbf{n}_n] = \mathbf{c}_n\mathbf{R}_{nn}\mathbf{c}_n^T + \sum_{k=1}^{N_b} \mathbf{L}_{nk}\mathbf{R}_{nn}\mathbf{L}_{nk}^T, \quad (33)$$

where $\mathbf{R}_{nn} = [r_{nn}[i-j]]$ for $i = 1, 2, \ldots, N_{ob} + N_d$ and $j = 1, 2, \ldots, N_{ob} + N_d$.

The term $\mathbf{z}_n$ results from the multiplication of the noise signal $\mathbf{n}_n$ with a delayed version of itself, making the derivation of its autocorrelation matrix complicated. The problem and its solution are well-known in the field of spectral analysis and estimation. For instance, [6] states that

$$\begin{aligned}&\mathcal{C}[n(t)n(t+u_1), n(\tau)n(t+\tau+u_2)]\\ &= r_{nn}(\tau)r_{nn}(\tau+u_2-u_1) + r_{nn}(\tau+u_2)r_{nn}(\tau-u_1), \quad (34)\end{aligned}$$

which, combined with an element-wise view on $\mathcal{A}[\mathbf{z}_n]$, reveals that

$$[\mathcal{A}[\mathbf{z}_n]]_{kl} = r_{nn}^2[k-l] + r_{nn}[k-l+N_d]r_{nn}[k-l-N_d]. \quad (35)$$

Since $r_{nn}[k]$ is an even function, the matrix $\mathcal{A}[\mathbf{z}_n]$ has a symmetrical Toeplitz structure, as one would expect for a stationary real valued noise signal like $n(t)n(t-d)$. This concludes the derivation of $\mathcal{A}[\mathbf{x}_n]$.

Successively, we will derive both the expectation and the autocovariance of the multiplier output conditioned on $\mathbf{b}_n$, relevant for the computation of the BEP. Their derivations resemble the derivation of their unconditional counterparts, such that only the results will be presented. The expectation for $\mathbf{x}_n$ conditioned on $\mathbf{b}_n$ is

$$\begin{aligned}\mathcal{E}[\mathbf{x}_n|\mathbf{b}_n] = &\mathbf{\Lambda}\left(\mathbf{R}_n^{(0)}\mathbf{1}\right)\mathbf{R}_n^{(d_n^{(1)})}\mathbf{1} + \mathbf{C}_n\mathbf{b}_n + \sum_{k=1}^{N_b}\mathbf{K}_{nk}\mathbf{b}_n[\mathbf{b}_n]_k\\ &+ r_{nn}[N_d]\mathbf{1}. \quad (36)\end{aligned}$$

The autocovariance matrix of $\mathbf{x}_n$ conditioned on $\mathbf{b}_n$ is

$$\mathcal{A}[\mathbf{x}_n|\mathbf{b}_n] = \mathcal{A}[\mathbf{z}_n] + \mathcal{A}[\mathbf{T}_n\mathbf{n}_n|\mathbf{b}_n], \quad (37)$$

where $\mathcal{A}[\mathbf{z}_n]$ is solved in Eq.(35). The second term is

$$\mathcal{A}[\mathbf{T}_n\mathbf{n}_n|\mathbf{b}_n] = \mathbf{c}_n\mathbf{R}_{nn}\mathbf{c}_n^T + \sum_{k,l=1}^{N_b}[\mathbf{b}_n]_k \mathbf{L}_{nk}\mathbf{R}_{nn}\mathbf{L}_{nl}^T [\mathbf{b}_n]_l \quad (38)$$

illustrating the dependency of the noise term $\mathbf{T}_n\mathbf{n}_n$ on $\mathbf{b}_n$.

B. Demodulator output

The derivation of the statistical characterization of α_n is straight-forward. The unconditional statistics of α_n are

$$\mathcal{C}\left[[\mathbf{b}_n]_{L_b}, \alpha_n\right] = \mathbf{w}_n^T\mathcal{C}\left[[\mathbf{b}_n]_{L_b}, \mathbf{x}_n\right], \quad (39)$$

$$\mathcal{E}[\alpha_n] = \mathbf{w}_n^T\mathcal{E}[\mathbf{x}_n] + o_n, \quad (40)$$

$$\mathcal{A}[\alpha_n] = \mathbf{w}_n^T\mathcal{A}[\mathbf{x}_n]\,\mathbf{w}_n \quad (41)$$

and their conditional counterpart are

$$\mathcal{E}[\alpha_n|\mathbf{b}_n] = \mathbf{w}_n^T\mathcal{E}[\mathbf{x}_n|\mathbf{b}_n] + o_n, \quad (42)$$

$$\begin{aligned}\mathcal{A}[\alpha_n|\mathbf{b}_n] &= \mathbf{w}_n^T\mathcal{A}[\mathbf{x}_n|\mathbf{b}_n]\,\mathbf{w}_n\\ &= \mathbf{w}_n^T\mathcal{A}[\mathbf{z}_n]\,\mathbf{w}_n + \mathbf{w}_n^T\mathcal{A}[\mathbf{T}_n\mathbf{n}_n|\mathbf{b}_n]\,\mathbf{w}_n. \quad (43)\end{aligned}$$

This finalizes the statistical characterization of the weighted correlation demodulator.

V. Computation of the BEP

In this section a method is described to estimate the BEP. Firstly, a deterministic channel is assumed for which we will compute the SEP. By repeating the SEP computation for many different randomly selected channels, the average SEP within a certain environment can be obtained. Since we assumed a deterministic channel, the BEP depends on the multi-user interference (MUI), the inter-symbol interference (ISI) and the thermal noise. From a mathematical perspective MUI and ISI are equivalent, which allowed us to collect the bits that cause ISI and MUI within $\mathbf{b}_n$. Additionally, $\mathbf{b}_n$ also contains the bit under demodulation. Let's define with $\mathbb{B}_a$ the set of bits that cause ISI and MUI given that the bit under demodulation has the value a, that is $\mathbb{B}_a = \{\mathbf{b}_n|\mathbf{b}_n \in \{-1,1\}^{N_b}; [\mathbf{b}_n]_{L_b} = a\}$.

The BEP conditioned on the bit vector $\mathbf{b}_n$ can be expressed using the Gaussian error function, whose argument depend on the useful signal, ISI, MUI and thermal noise, where we assumed that both noise terms in the demodulator output, are independent, Gaussian distributed RVs. These demodulator noise terms can be found on the right hand side of Eq.(43). The conditional error probability is

$$\mathrm{P}(e|\mathbf{b}_n) = \mathrm{Q}\left(\frac{c_0\mathcal{E}[\alpha_n|\mathbf{b}_n]}{\sqrt{\mathcal{A}[\alpha_n|\mathbf{b}_n]}}\right), \quad (44)$$

where $c_0 = 1$ if $\mathbf{b}_n \in \mathbb{B}_1$ and $c_0 = -1$ if $\mathbf{b}_n \in \mathbb{B}_{-1}$. Since all the realization of $\mathbf{b}_n$ have the same a-priori probability, the BEP is

$$\mathrm{P}(e) = 2^{-N_b}\sum\nolimits_{\mathbf{b}\in\mathbb{B}^{N_b}} \mathrm{P}(e|\mathbf{b}_n). \quad (45)$$

Eq.(45) points out that the BEP computation grows exponentially with N_b, making an exact computation of the BEP unfeasible even for systems of moderate size. In this case, we used the semi-analytical method described in [7].

VI. Weighting function

We have considered two types of weighting functions, the first minimizes the mean square error (LMMSE) and the second is based on maximum ratio combining (MRC), see ([8]). For both criteria, o_n is chosen such that the demodulator output is zero mean, that is

$$o_n = -\mathbf{w}_n^T\mathcal{E}[\mathbf{x}_n]. \quad (46)$$

According to the LMMSE rule, $\mathbf{w}_n$ is chosen such that

$$\mathbf{w}_n^L = \underset{\mathbf{w}_n\in\mathbb{R}^{N_{ob}}}{\operatorname{argmin}}\,\mathcal{E}\left[\left|[\mathbf{b}_n]_{L_b} - \mathbf{w}_n^T\mathbf{x}_n - o_n\right|^2\right]. \quad (47)$$

It can be shown that

$$\mathbf{w}_n^L = (\mathcal{A}[\mathbf{x}_n])^{-1}\mathcal{C}\left[[\mathbf{b}_n]_{L_b}, \mathbf{x}_n\right]. \quad (48)$$

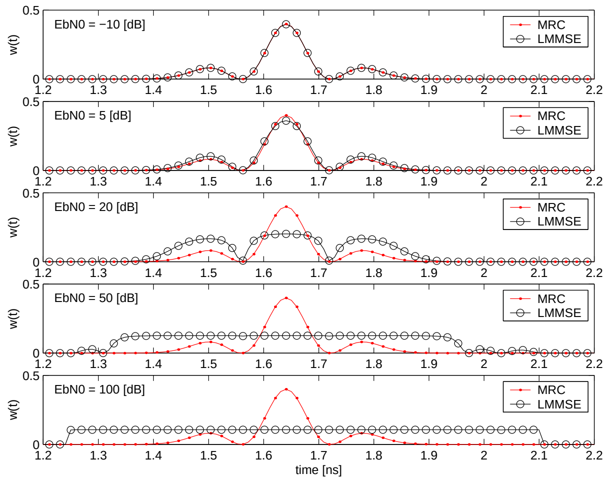

Fig. 2. The LMMSE and MRC weighting coefficient as function of E_b/N_0

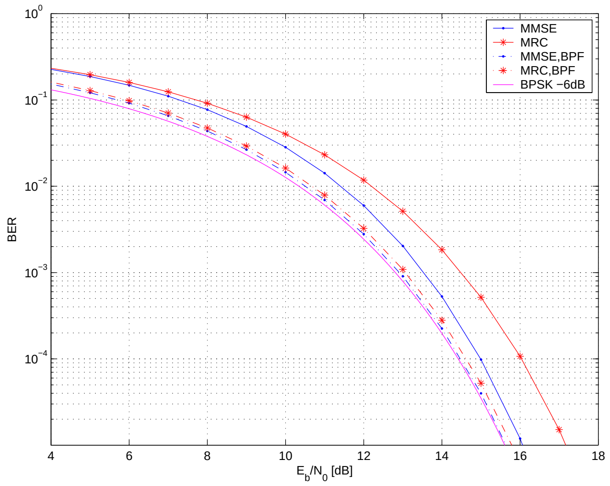

Fig. 3. The BEP performance of several DHTR systems on a single user, AWGN channel

The solution of the MRC rule becomes equivalent to the LMMSE one, whenever $\mathcal{A}[\mathbf{x}_n] \propto \mathbf{I}_{N_{ob},N_{ob}}$. The MRC weighting vector $\mathbf{w}_n^M$ minimizing Eq.(47) becomes

$$\mathbf{w}_n^M = c_1 \mathcal{C}\left[[\mathbf{b}_n]_{L_b}, \mathbf{x}_n \right], \tag{49}$$

where the value of c_1 itself has no impact on the BEP performance. Otherwise, the MRC strategy is sub-optimal in term of the LMMSE rule. The implementation of a MRC weighting is less complex, such that some degradation in performance is acceptable.

VII. Results

In this section, we present some basic results to illustrate the strength of the developed method. In Fig.2, LMMSE and MRC weighting coefficients are presented as function of E_b/N_0 for a system using the Gaussian monocycle on a single user, AWGN channel. No BPF is deployed and the noise has a bandwidth of 12.8 GHz, a result from the finite sampling rate. At low E_b/N_0, $n(t)n(t-D)$ is the dominant noise signal, such that $\mathbf{w}_n^L$ is almost identical to $\mathbf{w}_n^M$. With increasing E_b/N_0, $\mathbf{w}_n^L$ transform to an equal weighting strategy as deployed in [5], illustrating the optimality of an equal weighting scheme for the limiting case $E_b/N_0 \to \infty$. The BEP curves of both systems are depicted in Fig.(3). For reference purposes, the BPSK BEP is depicted on a AWGN channel with a 6 dB penalty. The BEP of the LMMSE system is lower than the BEP of the MRC system along the E_b/N_0 range. As to be expected from Fig.2, the difference vanishes when $E_b/N_0 \downarrow 0$. Additionally, we present the BEP of a system using a 3 GHz bandwidth, 4.5 GHz center frequency transmit pulse and a matched BPF at the receiver. As expected, the LMMSE weighting correlation receiver outperforms the MRC one at every E_b/N_0. However, this time the difference between the two structures is significantly smaller.

VIII. Conclusions

A generalization of the delay hopped transmit reference communication system has been proposed. A statistical characterization of the system has been derived together with a method to evaluate its BEP, taking into account the presence of multipath, inter-symbol interference, multi-user interference and for any type of front-end filter and weighting function. Two weighting criteria have been considered to determine the shape of the weighting function: the LMMSE and the MRC criteria. Their BEP have been presented for a single user, AWGN channel. The usage of the LMMSE rule leads to the best performance, while the MRC rule performs only slightly worse.

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